Can Single Dimpled Chads Reflect Voter Intent?

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It was reported on CNN that the Canvassing Board of Palm Beach County was counting dimpled chads as votes only when there were more than one dimpled chad on a ballot. The reasoning behind this seems intuitive: if all the other votes on the ballot were clearly perforated, then a single dimpled chad must not be an intended vote, but is more likely caused by a voter's decision not to finally make that vote.

Intuitively this might seem to make sense, and this has been argued by people objecting to the counting of dimpled chads. However, this reasoning is flawed. If on rare occasion a voter casting a vote accidentally dimples a chad instead of perforating it, then the vast majority of ballots that have dimpled chads will have exactly one dimpled chad, with the other votes on the ballot being clear perforations. Therefore, to restrict the count of dimpled chads only to those ballots that bear multiple dimpled chads will exclude the vast majority of intended votes that accidentally left only a dimple. This is explained mathematically below.

Analysis of Dimpled Chad Distributions

Suppose that each voter i has an error rate of ϵ_i of dimpling a chad instead of perforating the chad. Let the distribution of these error rates in the population be $f(\epsilon)$. Let the number of possible votes on a ballot be n. If each vote made by a given voter has the same chance, ϵ, of being a dimple instead of a perforation, independent of that voter's other votes, the the distribution of the number, k, of dimpled chads that will appear on that voter's ballot is:

$$V(k, \epsilon) = \binom{n}{k} \epsilon^k (1-\epsilon)^{n-k}$$

The distribution of the number of dimpled chads per ballot for the entire population is:

$$V(k) = \int_0^1 f(\epsilon)\ V(k,\epsilon)\ d\epsilon = \int_0^1 f(\epsilon)\ \binom{n}{k} \epsilon^k (1-\epsilon)^{n-k}\ d\epsilon$$

For concreteness, let us examine some specific examples.

# Dimpled Chads	Fraction of Ballots
0	0.904382
1	0.0913517
2	0.00415235
3	0.000111848
4	$1.97711\,10^{-6}$
5	$2.39649\,10^{-8}$
6	$2.01725\,10^{-10}$
7	$1.16436\,10^{-12}$
8	$4.41045\,10^{-15}$
9	$9.9\,10^{-18}$
10	$1.\,10^{-20}$

Table 1: The fraction of ballots having 0, 1, 2, etc. dimpled chads.

Example 1: A single error rate of voters

Suppose that all voters have the same error rate ϵ of producing a dimpled rather than a perforated chad. Let us suppose that the error rate is 1%. Suppose there are 10 items to be voted on on the ballot ($n = 10$). Then

$$V(k) = \binom{10}{k} 0.01^k 0.99^{10-k}$$

The values for the fraction of ballots that will have 0, 1, 2, etc. dimpled chads are given in Table 1. The key thing to note is that among all the ballots that have at least one dimpled chad ($0.0956179 = 1 - 0.904382$), fully 95.5% of them have only a single dimpled chad. This occurs under the assumption that dimpled chads are voter accidents when a vote was intended. Therefore, one does not need to have multiple dimpled chads on a ballot to be able to conclude that the dimpled chads were accidents.

Further examples for this analysis can be considered, and they all yield the same conclusion. The author would be happy to provide additional analysis or comment upon request.